NONISOTHERMAL EXTRUSION UNDER CONTINUOUS SHEAR OF LIQUIDS WITH AN ANOMALOUS VISCOSITY CHARACTERISTIC

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The flow of a liquid with an arbitrary anomalous viscosity–temperature characteristic through the channel of an extruder screw is analyzed taking account of the circulating flow, the energy dissipation, and the convective heat transfer.

The isothermal flow of a non-Newtonian liquid through the channel of an extruder screw has been studied in [1] under conditions of continuous shear. The liquid was assumed to behave according to a power-law rheological equation. The extruder channel was simulated by two parallel plates. It was possible to show the effect of the transverse circulating flow of the liquid on the overall flow characteristics and the extruder performance characteristics, namely the dependence of the productivity on the pressure head. It is even more important to take into account the transverse circulation when considering the problem of nonisothermal extrusion of highly viscous liquids. When the transverse circulation is taken into account, it becomes possible to determine more precisely the dissipation of energy, the dissipative heating of the extruded mass, and the pressure in the extruder head, and it becomes possible to calculate the power lost in the extruder which consists largely of dissipative heating of the extruded mass. For this reason, it will be worthwhile to solve the problem of continuous shear in the liquid under nonisothermal conditions when determining the thermal and mechanical characteristics of the extrusion process.

A nonisothermal shearing of a liquid the rheological behavior of which follows a power law has been analyzed by Griffith in [2]. That analysis was based on the assumption of complete thermal stabilization and, therefore, its results are applicable to long screws. Under real conditions there is not sufficient time for the temperature in the feed zone of the screw to stabilize and the heat transfer is thus effected mainly through convection.

In this article we analyze the nonisothermal flow of a liquid with an arbitrary anomalous viscosity–temperature characteristic through the channel of an extruder screw, taking into account the circulating flow as well as the energy dissipation and the convective heat transfer.

1. We will use a two-dimensional model of the channel and will analyze the motion of the liquid between two infinite parallel plates (Fig. 1). The distance between the plates is h. When the channel depth is small in comparison with the width, one may disregard the effect of the screw threads, one may assume the transverse flow profile to remain unchanged, and one may refer all calculations to unit channel width. Our analysis will be confined to the flow within a segment the length L of which corresponds to the length of the feed zone in a screw channel.

Let the velocity v_0 of the upper plate be resolved into two components along the x and y axes, respectively, and let pressure gradients $\partial p/\partial x$, $\partial p/\partial y$ act along the same axes but in opposite directions. Angle φ will correspond to the pitch angle of the screw.

At high flow velocities, when the transverse circulation produces a rapid displacement of mass, one may disregard the temperature variation over the channel depth and assume that it varies only along the

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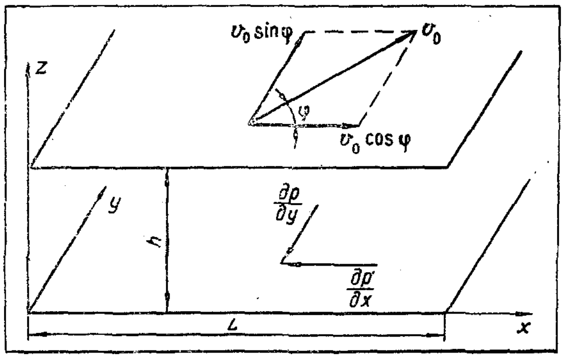

Fig. 1

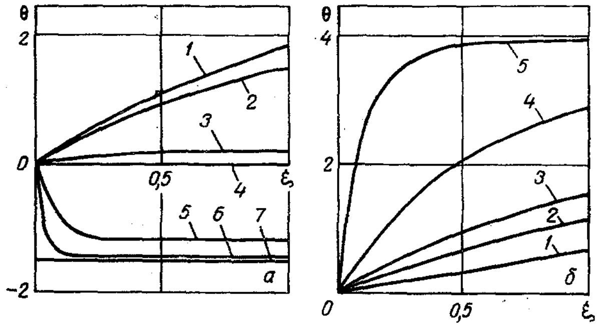

Fig. 2

Fig. 1. Two-dimensional model of the channel of an extruder screw.

Fig. 2. Effect of the heat-transfer coefficient α (cal/ $cm^2 \cdot sec \cdot deg$) and of the liquid flow rate on the temperature rise, as a function of the screw channel length. a: 1) $\alpha = 0$; 2) $2 \cdot 10^{-3}$; 3) $2 \cdot 10^{-2}$; 4) 0.174; 5) $2 \cdot 10^{-1}$; 6) 2; 7) ∞; b: 1) $Q_1 = 0.427$; 2) 0.25; 3) 0.174; 4) 0.073; 5) 0.01.

x-axis. The temperature at the entrance section at x = 0 will be denoted by T_e, the ambient temperature by T_0. The heat transfer between the liquid and the surrounding medium proceeds according to Newton's law with heat-transfer coefficient α.

The steady-state flow is laminar. The general form of the rheological equation for a liquid with an anomalous viscosity characteristic is

$$D_\varepsilon = k_0 f_1(T) f_2(H) D_\tau. \tag{1}$$

Here D_ε is the strain rate tensor; D_τ is the deviator of the stress tensor; and $k_0 f_1(T) f_2(H)$ is the reciprocal of the effective viscosity, a function of the temperature and the shear stress intensity; H is expressed in terms of shear stresses as

$$H = \sqrt{\tau_{xz}^2 + \tau_{yz}^2}. \tag{2}$$

The flow rate of the liquid is zero across the channel (because of the screw threads) and remains constant along the channel (law of continuity). Since the properties of this liquid are assumed to depend on the temperature, the forward and the transverse pressure gradient as well as the velocity profile are functions of x.

The equations of stress propagation are

$$\partial \tau_{xz}/\partial z = A_1, \ \partial \tau_{yz}/\partial z = A_2 \quad (A_1 = \partial p/\partial x, \ A_2 = \partial p/\partial y). \tag{3}$$

After integration, we have

$$\tau_{xz} = A_1 z + A_0 h C_1, \ \tau_{yz} = A_2 z + A_0 h C_2, \tag{4}$$

where A_0 is the pressure gradient along the x-axis at x = 0; C_1 and C_2 are integration constants (constant for a given section but different for different sections).

We now introduce the dimensionless quantities:

$$\zeta = z/h, \ \xi = x/L, \ v_1 = v_x/v_0, \ v_2 = v_y/v_0, \ a_1 = A_1/A_0, \ a_2 = A_2/A_0. \tag{5}$$

With (5), we obtain from (1) and (4):

$$\begin{aligned} \frac{\partial v_1}{\partial \zeta} &= \frac{k_0 A_0 h^2}{v_0} f_1(T) f_2(H)(a_1 \zeta + C_1), \\ \frac{\partial v_2}{\partial \zeta} &= \frac{k_0 A_0 h^2}{v_0} f_1(T) f_2(H)(a_2 \zeta + C_2). \end{aligned} \tag{6}$$

Changing to dimensionless variables in f_1 and f_2, we can rewrite (6) as

$$\frac{\partial v_1}{\partial \zeta} = \frac{F_1(\theta)}{\lambda} F_2(G)(a_1 \zeta + C_1), \ \frac{\partial v_2}{\partial \zeta} = \frac{F_1(\theta)}{\lambda} F_2(G)(a_2 \zeta + C_2). \tag{7}$$

Here θ is the dimensionless temperature; λ is a certain dimensionless group; $G = \sqrt{(a_1\zeta + C_1)^2 + (a_2\zeta + C_2)^2}$. The expressions for θ and λ, as well as for the functions F_1 and F_2, will depend on the specific form of functions f_1 and f_2. A specific example is considered below.

Equations (7) must be integrated with the following boundary conditions:

$$v_1 = v_2 = 0 \text{ for } \zeta = 0; \; v_1 = \cos\varphi, \; v_2 = \sin\varphi \text{ for } \zeta = 1. \tag{8}$$

Taking the first pair of conditions (8), we have

$$v_1 = \frac{F_1(\theta)}{\lambda}\int_0^\zeta F_2(G)(a_1\zeta + C_1)\,d\zeta, \quad v_2 = \frac{F_1(\theta)}{\lambda}\int_0^\zeta F_2(G)(a_2\zeta + C_2)\,d\zeta. \tag{9}$$

Expressions (9) contain the unknown quantities θ, a_1, a_2, C_1, and C_2. These can be determined from the equation of heat transfer, the second pair of boundary conditions (8), and the condition that the transverse flow rate is zero while the forward flow rate is Q_1. The expressions for the forward and the transverse flow rate in dimensionless units now become:

$$Q_1 = \int_0^1 v_1 d\zeta, \; Q_2 = \int_0^1 v_2 d\zeta. \tag{10}$$

Inserting v_1 and v_2 into (10) according to (9) yields

$$\begin{aligned} Q_1 &= \frac{F_1(\theta)}{\lambda}\int_0^1\int_0^\zeta F_2(G)(a_1\zeta + C_1)\,d\zeta d\zeta, \\ Q_2 &= \frac{F_1(\theta)}{\lambda}\int_0^1\int_0^\zeta F_2(G)(a_2\zeta + C_2)\,d\zeta d\zeta. \end{aligned} \tag{11}$$

Integrating (11) by parts will yield the following expressions for the flow rates:

$$\begin{aligned} Q_1 &= \frac{F_1(\theta)}{\lambda}\int_0^1 F_2(G)(a_1\zeta + C_1)(1-\zeta)\,d\zeta, \\ Q_2 &= \frac{F_1(\theta)}{\lambda}\int_0^1 F_2(G)(a_2\zeta + C_2)(1-\zeta)\,d\zeta. \end{aligned} \tag{12}$$

Satisfying the boundary conditions and also the conditions imposed on the flow rates in both directions, we arrive at the following system of equations:

$$\begin{aligned} &\frac{F_1(\theta)}{\lambda}\int_0^1 F_2(G)(a_1\zeta + C_1)\,d\zeta = \cos\varphi, \\ &\frac{F_1(\theta)}{\lambda}\int_0^1 F_2(G)(a_2\zeta + C_2)\,d\zeta = \sin\varphi, \\ &\frac{F_1(\theta)}{\lambda}\int_0^1 F_2(G)(a_1\zeta + C_1)\,\zeta d\zeta = \cos\varphi - Q_1, \\ &\frac{F_1(\theta)}{\lambda}\int_0^1 F_2(G)(a_2\zeta + C_2)\,\zeta d\zeta = \sin\varphi. \end{aligned} \tag{13}$$

The last two equations in (13) have been obtained with the aid of the first two.

We now proceed to the equation of heat transfer. For unit length of the screw channel, this equation can be written as:

$$c\rho \frac{\partial T}{\partial x} \int_0^h v_x dz = \frac{1}{J} \int_0^h \left(\tau_{xz} \frac{\partial v_x}{\partial z} + \tau_{yz} \frac{\partial v_y}{\partial z} \right) dz - \alpha (T - T_0). \tag{14}$$

Interest attaches to the actual temperature rise $T - T_e$ of the liquid and we introduce into the analysis a dimensionless temperature rise $(T - T_e)/M$, where M is some heating scale factor chosen on the basis of the relation $f_1(T)$. Changing to dimensionless variables, we obtain

$$\frac{c\rho M v_0 h}{L} \cdot \frac{d\theta}{d\xi} \int_0^1 v_1 d\zeta = \frac{A_0 h v_0}{J} \int_0^1 \left[(a_1\zeta + C_1) \frac{\partial v_1}{\partial \zeta} + (a_2\zeta + C_2) \frac{\partial v_2}{\partial \zeta} \right] d\zeta - \alpha M\theta - \alpha (T_e - T_0). \tag{15}$$

The integral on the left-hand side of Eq. (15) represents the dimensionless forward flow rate and is equal to Q_1. The integral on the right-hand side of (15) can, with the aid of (7) and (13), be written as

$$\int_0^1 \left[(a_1\xi + C_1) \frac{\partial v_1}{\partial \zeta} + (a_2\zeta + C_2) \frac{\partial v_2}{\partial \zeta} \right] d\zeta = (a_1 + C_1) \cos\varphi + (a_2 + C_2) \sin\varphi - a_1 Q_1. \tag{16}$$

If we denote the dimensionless groups as follows:

$$\varkappa = \frac{A_0 L}{J c\rho M Q_1}, \quad \mu = \frac{\alpha L}{c\rho v_0 h Q_1}, \quad \nu = \frac{\alpha L (T_0 - T_e)}{c\rho v_0 h M Q_1},$$

then the final equation of heat balance and its boundary conditions will be

$$\frac{d\theta}{d\xi} = \varkappa \left[(a_1 + C_1) \cos\varphi + (a_2 + C_2) \sin\varphi - a_1 Q_1 \right] - \mu\theta + \nu,$$
$$\theta = 0 \quad \text{for} \quad \xi = 0. \tag{17}$$

It is interesting to note the following. The work of internal friction forces in the liquid in the screw channel can be calculated in two ways: approximately and exactly. In the first case, the work W per unit time is defined as the product of the shear stress at the boundary and the corresponding screw velocity. In the second case, W is expressed as the integral of the product of shear stresses and shear rates over the entire channel depth (left-hand side of Eq. (16)). The expression $(a_1 + C_1) \cos\varphi + (a_2 + C_2) \sin\varphi$ on the right-hand side of Eq. (16) is no other than the scalar product of the stress vector and the shear rate vector at the boundary, i.e., the work of external forces. In this way, as (16) indicates, the value obtained for the work of viscous forces by the approximate method is too high. The magnitude of this error depends on the pressure gradient and on the liquid flow rate. Generally, from the right-hand side of (16) one must still subtract the product a_2Q_2, where Q_2 is the transverse flow rate. Both methods of determining the dissipative forces become equivalent only in the case when the extruder exit is either completely open or completely shut, i.e., when either the pressure gradient or the flow rate is zero. The maximum divergence between the two methods occurs at some intermediate mode of extruder operation.

Using the boundary condition, one can rewrite the integral of (17) as

$$\int_0^\theta \frac{d\theta}{\varkappa \left[(a_1 + C_1) \cos\varphi + (a_2 + C_2) \sin\varphi - a_1 Q_1 \right] - \mu\theta + \nu} = \xi. \tag{18}$$

In this way, with a definite value assigned to ξ, we have now a system of five equations (13) and (18) for determining the five unknowns θ, a_1, a_2, C_1, and C_2. In other words, the problem of determining the temperature, the pressure gradient, and the velocity profile at every section reduces to solving this system of five transcendental equations.

Integrating the forward pressure gradient over the channel length, we obtain the total pressure corresponding to the pressure difference between extruder entrance and exit:

$$\Delta p = A_0 L \int_0^1 a_1 d\xi. \tag{19}$$

Analogously we obtain an expression for the total power dissipated on heating the liquid, per unit clearance width:

$$N = A_0 L h v_0 \int_0^1 \left[(a_1 + C_1)\cos\varphi + (a_2 + C_2)\sin\varphi - a_1 Q_1\right] d\xi. \tag{20}$$

As has been noted earlier, the quantity A_0 is the pressure gradient at the entrance section x = 0 ($\xi = 0$). In order to determine A_0, it is sufficient to let $\theta = 0$ and $a_1 = 1$ in Eqs. (13), where A_0 appears implicitly in the λ-group, and then to solve (13) for A_0, a_0, C_1, and C_2.

2. In order to illustrate this theoretical analysis and to explore the effect of various parameters on the temperature rise of the liquid, we have performed calculations for a specific kind of liquid with a definite viscosity–temperature characteristic. It was assumed that the rheological behavior of the liquid could be described by a power-law equation with a Reynolds relation between viscosity and temperature. In this case the expressions for functions f_1, f_2, F_1, and F_2, parameter λ, and the heating scale factor M become

$$f_1(T) = \exp\left[\frac{\beta}{n}(T - T_e)\right], \quad f_2(H) = H^{\frac{1-n}{n}}, \quad F_1(\theta) = \exp\theta,$$

$$F_2(G) = G^{\frac{1-n}{n}}, \quad \lambda = \frac{v_0}{k_0 h (A_0 h)^{\frac{1}{n}}}, \quad M = \frac{n}{\beta}.$$

The following values were assigned to the parameters: h = 0.5 cm, L = 200 cm, $\varphi = 20°$, $v_0 = 20$ cm/sec; n = 0 4, $k_0 = 10^{-6}$ $(\mathrm{cm}^2/\mathrm{g})^{1/n} \cdot \mathrm{sec}^{-1}$, $J = 4.27 \cdot 10^4$ g·cm/cal, c = 1 cal/g·deg, $\rho = 1.65$ g/cm^3, $\beta = 0.02$ deg^{-1}, $T_e = 343$°K, $T_0 = 313$°K, $Q_1 = 0.174$, and $\alpha = 2 \cdot 10^{-3}$ cal/cm^2·sec·deg.

The system of equations consisting of Eqs. (13) and (18) has been solved numerically on a computer. The results are shown in Figs. 2 and 3.

In Fig. 2a the dimensionless temperature rise θ is shown as a function of the dimensionless channel length at various values of the heat-transfer coefficient α. Curve 1 corresponds to an adiabatic flow ($\alpha = 0$). If the channel of the extruder screw is lengthened infinitely, then the temperature rise will increase infinitely and the slope of the tangent to curve 1 will decrease monotonically toward zero at an infinite distance from the screw entrance. A decrease of this slope is associated with a lower intensity of heat generation, on account of the effective viscosity decreasing with higher temperature.

At values of α different from zero the curves level off, i.e., the temperature rise approaches a certain finite value, this value becoming lower as α increases If $T_0 < T_e$, then the temperature rise may become negative at sufficiently high values of α (curves 5, 6, 7), i.e., the liquid cools down as it moves through the channel. Under the extreme condition, when $\alpha \to \infty$, the flow becomes isothermal: all the heat generated at any section of the moving liquid is instantaneously carried away outside (curve 7). Thus, the temperature of the liquid will then be everywhere equal to the ambient temperature.

It is noteworthy that, if $T_0 < T_e$, there is such a value of the heat-transfer coefficient $\alpha = \alpha_*$ at which the maximum temperature rise and, consequently, also the overall temperature rise are zero, i.e., at which the temperature of the liquid does not change along the entire channel length (curve 4). This value of $\alpha = \alpha_*$ can be found from Eqs. (13) and (17) by letting $\theta = 0$ and $d\theta/d\xi = 0$ before solving for α, a_1, a_2, C_1, and C_2.

In order to evaluate how the departure of the given liquid from an ideal Newtonian liquid affects the temperature rise, we have calculated the temperature rises at $\alpha = 2 \cdot 10^{-3}$ cal/cm^2·sec·deg, $Q_1 = 0.2$, and with various values of the power exponent n. The trend of the curves is the same as before; they monotonically approach a maximum level. As n is decreased, the calculated temperature rise of the liquid becomes smaller, which can be explained by the lower effective viscosity:

$$\mu_{\mathrm{eff}} = \frac{1}{k_0}\exp\left[-\frac{\beta}{n}(T - T_e)\right] H^{1-\frac{1}{n}}.$$

All this is valid for pseudoplastic liquids. In the case of dilatant liquids, the temperature rise will increase as n is made other than unity.

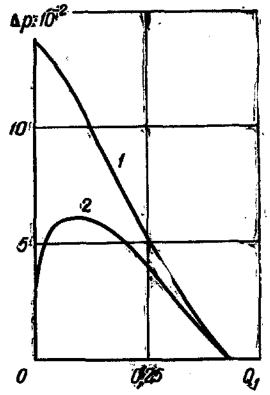

Fig. 3

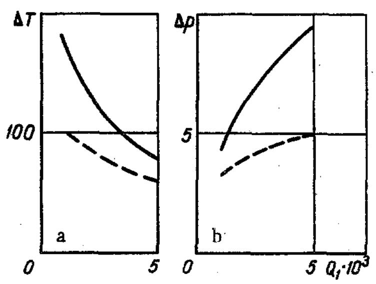

Fig. 4

Fig. 3. Pressure drop ΔP (kg/cm^2) as a function of the flow rate: isothermal flow (1) and nonisothermal flow (2).

Fig. 4. Comparison between theoretical values and experimental data: ΔT (°C) and Δp (kg/cm^2).

In Fig. 2b the effect is shown of various extruder exit conditions on the θ vs ξ relation at $\alpha = 2 \cdot 10^{-3}$ cal/cm$^2 \cdot$ sec $\cdot$ deg and n = 0.4. Curve 1 corresponds to a completely open exit. As the exit is being shut (Q_1 decreasing), the temperature of the mass rises, the curves become steeper and level off sooner. When the exit is completely shut, there occurs no convective heat transfer and the temperature rise of the mass becomes the same over the entire channel length. It can be determined from Eqs. (13) and (17) with $Q_1 = 0$ and $d\theta/d\xi = 0$.

The analysis of isothermal extrusion in [1] has established that, when the velocity at the boundary remains constant (constant rpm of the screw), the pressure gradient and, consequently, the pressure difference Δp between extruder entrance and exit both increase monotonically as the flow rate decreases until they reach their maximum values when the exit is completely shut. During nonisothermal extrusion the flow pattern may be qualitatively different. The curves in Fig. 3 represent the pressure difference Δp as a function of the flow rate Q_1 for isothermal extrusion (curve 1) and for nonisothermal extrusion (curve 2). It can be seen here that, at the given parameter values, the pressure difference first increases with a decreasing flow rate during nonisothermal extrusion, until it reaches its maximum value at some flow rate, and then continues to decrease. The explanation for this decreasing Δp is that, when the extruder exit is almost completely shut under certain conditions (high rpm of the screw and high viscosity of the liquid), the temperature of the mass rises fast and its effective viscosity drops sharply. The result is a decrease in the local pressure gradients along the major portion of the screw and, therefore, a lower overall pressure drop.

3. The foregoing analysis was based on the two-dimensional model of an extruder. In order to assess the applicability of the derived formulas for calculating the characteristics of a real extruder, we have compared the theoretical results with experimental data pertaining to a synthetic-fiber extrusion at the VNII laboratory (in Kalinin). The data here apply to the acrylonitrile copolymer with methylacrylate in dimethylformamide. The thermophysical characteristics of this liquid were determined, its rheological equation of flow was evaluated, and the screw channel was replaced by a two-dimensional model, whereupon the following values were obtained for the basic parameters: h = 0.285 cm, L = 30.8 cm, $\varphi = 38°$, $v_0 = 827$ cm/sec, n = 0.477, $k_0 = 0.65$ (cm^2/g)$^{1/n}$ sec^{-1}, c = 0.491 cal/g $\cdot$ deg, $\rho = 1$ g/cm^3, $\beta = 0.0137$ deg^{-1}, $T_e = T_0 = 295$°K, and $\alpha = 0$.

The temperature rise and the pressure drop (in dimensional units) have been plotted in Fig. 4 as functions of the flow rate. The test curves are dashed, the theoretical curves are solid Since the adiabatic flow was assumed in the calculations ($\alpha = 0$), the solid curve in Fig. 4a yields higher values. The trend of both curves is qualitatively the same. Within this given range of flow rates the two curves differ at most by 75% and at least by 28%. The curves tend to converge as the flow rate increases. Considering that the extruder operated here with the exit almost shut (the flow rates in this range amounted to less than 2% of the maximum possible flow rate), a closer agreement between theoretical and experimental values can be expected in the intermediate modes of extruder operation. Indeed, additional tests have shown that changing the flow rate from 15 to 45% maximum will decrease the maximum difference between tested and calculated values to 10% at most. Thus, within the range of flow rate variation most often encountered in practice the agreement is entirely satisfactory.

The trend of the curves in Fig. 4b confirms the validity of our earlier conclusions concerning the existence of extruder operating modes where a drop in the pressure is observed during a reduction of the flow rate. The explanation for the theoretical values being consistently higher than the experimental ones is that the clearance between the screw thread and the extruder case had been disregarded in the theoretical calculations. Leakage of liquid through this clearance can, according to tests, considerably lower the pressure drop along the channel.

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NOTATION

x, y, z	are Cartesian coordinates;
h	is the depth of the screw channel;
L	is the length of the screw channel;
φ	is the pitch angle of the screw flight;
v_0	is the velocity of the upper plate;
β, n, k_0	are rheological constants;
c	is the specific heat;
ρ	is the density;
J	is the mechanical equivalent of heat;
T	is the temperature;
p	is the pressure;
A_1, A_2	are pressure gradients;
D_ε	is the strain rate tensor;
D_τ	is the deviator of the stress tensor;
τ_{xz}, τ_{yz}	are stress tensor components;
M	is the heating scale factor;
W	is the work;
N	is the power;
α	is the heat-transfer coefficient;
Q_1	is the dimensionless flow rate;
λ, $\varkappa$, μ, ν	are dimensionless parameters.

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